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NON-ARCHIMEDEAN FUZZY MENGER SPACES

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Abstract: In this paper we study Random Probabilistic metric space known as non-Archimedean Random Probabilistic metric space. Our object in this section is to study on fixed points in non-Archimedean Random Probabilistic Space for quasi-contraction type pair .

Keywords: -Archimedean random probabilistic metric spaces, Cauchy Sequence, Random Fixed Point.
AMS Classification: 47H10, 54H25.

Introduction

Istrătescu and Crivăt [13] first studied Non-Archimedean probablistic metric spaces and some topological preliminaries on them. Achari [1] studied the fixed points of quasi-contraction type mappings in non-Archimedean PM-spaces and generalized the results of Istrătescu [12]. The fundamental results of Sehgal-Bharucha-Reid [17], Sherwood [18], were generalized and extended by many authors out of which some prominent one's are Achari [1], Chang [2, 3, 4, 5], Chang S.S. and Huang, N. J. [6], Ciric [7], Hadzic [8, 9, 10, 11], Istrătescu and Sacuiu [14], Vasuki [24], Mishra S.N.,Singh S.L. and Talwar R. [16], Singh & Pant [19,20,21,22,23] etc.

2. PRELIMINARIES:

Now we recall some well know established definitions :-

Definition 2.1: A Random probabilistic metric space is an ordered pair (ΩxX,F) where X is a nonempty set, L be set of all random distribution function and $F:(\Omega xX)\times(\Omega xX)\rightarrow L$ (collection of all random distribution functions). The value of $F((\omega,x),(\omega,y))$ at $(\omega\,,u)\in(\Omega xX)\times(\Omega xX)$ is represented by $F((\omega,x),(\omega,y))(\omega\,,u)$ satisfy the following conditions:

$[RPM-1]F((\omega,x),(\omega,y))(\omega\,,u)\ =\omega$ if and only if $(\omega,x)=(\omega,y)$

$[RPM-2].\,F((\omega,x),(\omega,y))(\omega)=(\omega)$ for every $(\omega,x),(\omega,y)\in\Omega xX$

$[RPM-3].\,F((\omega,x),(\omega,y))(\omega\,,u)=\ F((\omega,y),(\omega,x))(\omega\,,u)$

for every $(\omega,x),(\omega,y)\in\Omega xX$

$[RPM-4].\,F((\omega,x),(\omega,y))(\omega\,,u)\ =\omega$ and $F((\omega,y),(\omega,z))(\omega\,,v)=\omega$

then $F((\omega,x),(\omega,z))((\omega\,,u)+(\omega,v)\ =\omega$

for every $(\omega,x),(\omega,y),(\omega,z)\in\Omega xX.$

A Random Probabilistic Metric Space $(\Omega xX,F)$ is called non-Archimedean RPM – space if it satisfies

$[RPM-5].\,F((\omega,x),(\omega,y))(\omega\,,u)\ =\omega$ and $F((\omega,y),(\omega,z))(\omega\,,v)=\omega$

then $F((\omega,x),(\omega,z))\max((\omega\,,u),(\omega,v))\ =\omega$

for every $(\omega,x),(\omega,y),(\omega,z)\in\Omega xX$ instead of $[RPM-4]$.

Definition 2.2: A mapping $T:\ [0,1]\times[0,1]\ \rightarrow\ [0,1]$ is called t-norm if

1. $T\,(a,1)=\ a,\forall\, a\in[0,1]$
2. $T(0,0)\ =\ 0,$

3. $T(a,b) = T(b,a)$,
4. $T(c,d) \geq T(a,b)$ for $c \geq a, d \geq b$, i.e. T is non-decreasing in both co-ordinates
5. $T(T(a,b),c) = T\big(a, T(b,c)\big) \forall\, a,b,c \in [0,1]$
i.e. T is associative.

Definition 2.3: In addition of definition 2.1, if T is continuous on $[0,1] \times [0,1]$ and $T(a,a) < a, a \in [0,1]$, then T is called an Archimedean t – norm. A characterization of Archimedean t – norm is due to Ling [15]. He proved that a t – norm T is Archimedean if and only if it admits the representation,
$T(a,b) = g^{-1}\,[g(a) + g(b)]$
where g is continuous and decreasing function from $[0,1]$ to $[0,\infty]$ with $g(1) = 0$ and $g(0) = \infty$ and g^{-1} is the pseudo inverse of g, (c.f. Chang [2]$(g \circ g^{-1})(a) = a$, for all a in the range of g.
The continuous decreasing function g appearing in this characterization is called an additive generator of the Archimedean t-norm T.

Definition 2.4: A non-Archimedean Random Probabilistic space is an ordered triplet (ΩxX,F,T) where (ΩxX, F) is non- Archimedean RPM-space, T is a t-norm with the non-Archimedean triangle inequality;
$F\big((\omega,x),(\omega,z)\big)\max\{(\omega,u),(\omega,v)\}$
$\geq T\{F\big((\omega,x),(\omega,y)\big)(\omega,u), F\big((\omega,y)\big),(\omega,z)(\omega,v)\}$
as $F\big((\omega,x),(\omega,y)\big)(\omega,u) = \omega$ and $F\big((\omega,y),(\omega,z)\big)(\omega,v) = \omega$
then $F\big((\omega,x),(\omega,z)\big)\max((\omega,u),(\omega,v)) = \omega$

Definition 2.5: (Achari[1]) Let (ΩxX, F, T) be a non- Archimedean Random Probabilistic Metric space and let g be an additive generator, and let α be a number such that $0 < \alpha < 1$. A mapping $T\colon \Omega xX \to \Omega xX$ is a quasi – contraction mapping on ΩxX with respect g and α if for every $(\omega,x),(\omega,y) \in \Omega xX$

$$g\{F\big(T(\omega,x),T(\omega,y)\big)(\omega,u)\} \leq \alpha g \max\left\{\begin{matrix} F\big((\omega,x),(\omega,y)\big)\left(\dfrac{(\omega,u)}{\alpha}\right), F\big((\omega,x),T(\omega,x)\big)\left(\dfrac{(\omega,u)}{\alpha}\right), \\ F\big((\omega,y),T(\omega,y)\big)\left(\dfrac{(\omega,u)}{\alpha}\right)\end{matrix}\right\}$$

Definition 2.6: (Achari[1]) A mapping T: ΩxX → ΩxX is a quasi-contraction type map on a non-Archimedean RPM-space (ΩxX, F) if and only if there exists a constant $\alpha \in (0,1)$ such that

$$F\big(T(\omega,x),T(\omega,y)\big)(\omega,u) \leq \max\left\{\begin{matrix} F\big((\omega,x),(\omega,y)\big)\left(\dfrac{\omega,u}{\alpha}\right), F\big((\omega,x),T(\omega,x)\big)\left(\dfrac{\omega,u}{\alpha}\right), \\ F\big((\omega,y),T(\omega,y)\big)\left(\dfrac{\omega,u}{\alpha}\right)\end{matrix}\right\}$$

for all $(\omega,x),(\omega,y) \in \Omega xX$ and $(\omega,u) > 0, 0 < \alpha < 1$.
This can be interpreted as the random probability that the distance between the image points $T(\omega,x), T(\omega,y)$ is less than (ω,u) is at least equal to the probability that the maximum distances between $(\omega,x),(\omega,y)$ and $(\omega,x), T(\omega,x)$ and $(\omega,y), T(\omega,y)$ is less than (ω,u).

Definition 2.7: Let (ΩxX, F) be a non-Archimedean RPM-Space and let g be an additive generator, and let α be a number such that $0 < \alpha < 1$. The mappings G, $T\colon \Omega xX \to \Omega xX$ is called a quasi-contraction type pair of mappings on X with respect g and α if for every $(\omega,x),(\omega,y) \in \Omega xX$ and $(\omega,u) > 0$,

$$g\{F(G(\omega,x),T(\omega,y))(\omega,u)\} \leq \alpha g \max\left\{\begin{matrix} F\big((\omega,x),(\omega,y)\big)\left(\dfrac{(\omega,u)}{\alpha}\right), F\big((\omega,x),G(\omega,x)\big)\left(\dfrac{(\omega,u)}{\alpha}\right), \\ F((\omega,y),T(\omega,y))\left(\dfrac{(\omega,u)}{\alpha}\right)\end{matrix}\right\}$$

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Definition 2.8: Mappings $G,T:\Omega xX \to \Omega xX$ is a quasi- contraction type pair of maps on a non-Archimedean RPM-space $(\Omega xX, F)$ if and only if there exists a constant $\alpha \in (0,1)$ such that

$$\{F(G(\omega,x),T(\omega,y))(\omega,u)\} \le \max \left\{ \begin{array}{c} F((\omega,x),(\omega,y))\left(\frac{(\omega,u)}{\alpha}\right), F((\omega,x),G(\omega,x))\left(\frac{(\omega,u)}{\alpha}\right), \\ F((\omega,y),T(\omega,y))\left(\frac{(\omega,u)}{\alpha}\right) \end{array} \right\}$$

for all $(\omega,x),(\omega,y) \in \Omega xX$ and $(\omega,u) > 0, 0 < \alpha < 1$.

3. MAIN RESULT:

We establish fixed point theorems for a quasi-contraction type pair and a triplet of maps on complete non-Archimedean Random Probabilistic space.

Theorem 3.1: Let $(\Omega xX, F, T)$ be a non Archimedean Random Probabilistic Space under the Archimedean t-norm T, with the additive generator g. Let G and T be two self mappings of ΩxX into itself satisfying;

$$(3.1(a))\, g\{F(G(\omega,x),T(\omega,y))(\omega,u)\} \le \alpha g \max \left\{ \begin{array}{c} F((\omega,x),(\omega,y))\left(\frac{(\omega,u)}{\alpha}\right), F((\omega,x),G(\omega,x))\left(\frac{(\omega,u)}{\alpha}\right) \\ F((\omega,y),T(\omega,y))\left(\frac{(\omega,u)}{\alpha}\right), F((\omega,y),G(\omega,x))\left(\frac{(\omega,u)}{\alpha}\right), \end{array} \right\}$$

for all $(\omega,x),(\omega,y) \in \Omega xX$ and $(\omega,u) > 0, 0 < \alpha < 1$.

(3.1(b)) G and T are continuous on ΩxX.

Then G and T have a unique common fixed point in ΩxX.

Proof: Let $(\omega,x_0) \in \Omega xX$ be a arbitrary element and $\{(\omega,x_n)\}$ be a sequence in X such that

$(\omega,x_{2n+1}) = G(\omega,x_{2n}), (\omega,x_{2n+2}) = T(\omega,x_{2n+1}),\quad n = 0,1,2,3 \ldots\ldots\ldots$

be the sequence of iterates under the pair {G, T} at (ω,x_0).

Now from (3.1(a))

$$g\{F(\omega,x_1),(\omega,x_2)(\omega,u)\} = g\{FG(\omega,x_0),T(\omega,x_1)(\omega,u)\}$$

$$\le \alpha g \max \left\{ \begin{array}{c} F((\omega,x_0),(\omega,x_1))\left(\frac{(\omega,u)}{\alpha}\right), F((\omega,x_0,G(\omega,x_0))\left(\frac{(\omega,u)}{\alpha}\right), \\ F((\omega,x_1),T(\omega,x_1))\left(\frac{(\omega,u)}{\alpha}\right), F((\omega,x_1),G(\omega,x_0))\left(\frac{\omega,u}{\alpha}\right) \end{array} \right\}$$

$$\le \alpha g \max \left\{ \begin{array}{c} F((\omega,x_0)(\omega,x_1))\left(\frac{(\omega,u)}{\alpha}\right), F((\omega,x_{0,})(\omega,x_1))\left(\frac{(\omega,u)}{\alpha}\right), \\ F((\omega,x_1),(\omega,x_2))\left(\frac{(\omega,u)}{\alpha}\right), F((\omega,x_1),(\omega,x_1))\left(\frac{(\omega,u)}{\alpha}\right) \end{array} \right\}$$

$$\le \alpha g \left\{F((\omega,x_0),(\omega,x_1))\left(\frac{(\omega,u)}{\alpha}\right)\right\}$$

$$g\left\{F((\omega,x_1),(\omega,x_2))\left(\frac{(\omega,u)}{\alpha}\right)\right\} \le \alpha g\left\{F((\omega,x_0),(\omega,x_1))\left(\frac{(\omega,u)}{\alpha}\right)\right\}$$

Again,

$$g\left\{F((\omega,x_2),(\omega,x_3))\left(\frac{(\omega,u)}{\alpha}\right)\right\} \le g\left\{F((\omega,x_1),(\omega,x_2))\left(\frac{(\omega,u)}{\alpha}\right)\right\}$$

$$\leq \alpha g\left\{F((\omega,x_1),(\omega,x_2))\left(\frac{(\omega,u)}{\alpha}\right)\right\}$$
$$\leq \alpha^2 g\left\{F((\omega,x_0),(\omega,x_1))\left(\frac{(\omega,u)}{\alpha^2}\right)\right\}$$

Therefore,

$$g\left\{F((\omega,x_2),(\omega,x_3))\left(\frac{(\omega,u)}{\alpha}\right)\right\} \leq \alpha^2 g\left\{F((\omega,x_0),(\omega,x_1))\left(\frac{(\omega,u)}{\alpha^2}\right)\right\}$$

Hence it follows by induction that for every positive integer n,

$$g\left\{F((\omega,x_n),(\omega,x_{n+1}))\left(\frac{(\omega,u)}{\alpha}\right)\right\} \leq \alpha^n g\left\{F((\omega,x_0),(\omega,x_1))\left(\frac{(\omega,u)}{\alpha^2}\right)\right\} \dots\dots..(3.1.1)$$

Now for $m > n > 0$ and $(\omega,u) > 0$ we have,

$$F((\omega,x_{2n+1}),(\omega,x_{2n+2m}))\left(\frac{(\omega,u)}{\alpha}\right) \geq T\{F((\omega,x_{2n+1}),(\omega,x_{2n+2}))\left(\frac{(\omega,u)}{\alpha}\right),, F((\omega,x_{2n+2}),(\omega,x_{2n+2m}))\left(\frac{(\omega,u)}{\alpha}\right)\}$$
$$F((\omega,x_{2n+1}),(\omega,x_{2n+2}))(\omega,u), F((\omega,x_{2n+2}),(\omega,x_{2n+2m}))(\alpha(\omega,u))\}$$

Since $\alpha < 1$ and T is non decreasing and (RPM-5)

$$F((\omega,x_{2n+1}),(\omega,x_{2n+2m}))(\omega,u)$$
$$\geq T\left\{\begin{matrix} F((\omega,x_{2n+1}),(\omega,x_{2n+2}))(\omega,u), F((\omega,x_{2n+1}),(\omega,x_{2n+2}))(\alpha(\omega\, u)), \\ , F((\omega,x_{2n+2}),(\omega,x_{2n+2m}))(\alpha^2(\omega,u))) \end{matrix}\right\}$$
$$\geq T\left\{\begin{matrix} T(F((\omega,x_{2n+1}),(\omega,x_{2n+2}))(\omega,u), F((\omega,x_{2n+2}),(\omega,x_{2n+3}))(\alpha(\omega,u)), \\ F((\omega,x_{2n+2}),(\omega,x_{2n+2m}))(\alpha^2(\omega,u)) \end{matrix}\right\}$$
$$= g^{-1}\{g[T(F((\omega,x_{2n+1}),(\omega,x_{2n+2}))(\omega,u), F((\omega,x_{2n+2}),(\omega,x_{2n+3}))(\alpha(\omega,u))] + g[F((\omega,x_{2n+2}),(\omega,x_{2n+2m}))(\alpha^2(\omega,u))]\}$$

$$= g^{-1}\{g[g^{-1}\{g[F((\omega,x_{2n+1}),(\omega,x_{2n+2}))(\omega,u)] + g[F((\omega,x_{2n+2}),(\omega,x_{2n+3}))(\alpha(\omega,u))]\}] + g[F((\omega,x_{2n+2}),(\omega,x_{2n+2m}))(\alpha^2(\omega,u))]\}$$

$$\geq g^{-1}\left\{\begin{matrix} g\left[g^{-1}\left\{\begin{matrix}\alpha^{2n+1} g\left[F((\omega,x_0)(\omega,x_1))\left(\frac{(\omega,u)}{\alpha^{2n+1}}\right)\right] \\ +\alpha^{2n+2} g\left[F((\omega,x_0)(\omega,x_1))\left(\frac{(\omega,u)}{\alpha^{2n+1}}\right)\right]\end{matrix}\right\}\right] \dots\dots \\ +g[F((\omega,x_{2n+2m-1})(\omega,x_{2n+2m}))(\alpha^{2m-2}\,(\omega,u)] \end{matrix}\right\}$$

$$\geq g^{-1}\left\{\begin{matrix} g\left[g^{-1}\left\{\begin{matrix}\alpha^{2n+1} g\left[F((\omega,x_0)(\omega,x_1))\left(\frac{(\omega,u)}{\alpha^{2n+1}}\right)\right] \\ +\alpha^{2n+2} g\left[F((\omega,x_0)(\omega,x_1))\left(\frac{(\omega,u)}{\alpha^{2n+1}}\right)\right]\end{matrix}\right\}\right] \dots\dots \\ +\alpha^{2n+2m+2} g[F((\omega,x_0)(\omega,x_1))\left(\frac{(\omega,u)}{\alpha^{2n+1}}\right)\left(\frac{(\omega,u)}{\alpha^{2n+1}}\right)] \end{matrix}\right\}$$

Hence we conclude $\{(\omega,x_n)\}$ is a Cauchy sequence, since g^{-1} and g are continuous, $\alpha \to 0$, as $n \to \infty$, $F((\omega,x),(\omega,y))(\omega,u) \to \omega$ as $(\omega,u) \to \infty$ and $g^{-1}(0) = \omega$.Since $((\Omega x X), F, T)$ is complete there is point $(\omega,z) \in \Omega\, xX$ such that $(\omega,x_n) \to (\omega,z)$.

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According to Istrătescu and Sacuiu [8], the subsequences $\{(\omega, x_n)\}$, $\{(\omega, x_{n+1})\}$ converges to (ω, z) i.e. $(\omega, x_n \to (\omega, z)$, $(\omega, x_{n+1}) \to (\omega, z)$ continuity of G and T implies $G(\omega, x_n) \to G(\omega, z), T(\omega, x_n) \to T(\omega, z)$.
We shall now show that (ω, z) is common fixed point of G and T.
However we have,

$$F((\omega,z),G(\omega,z))(\omega,u) \geq T\left\{\begin{matrix} F\big((\omega,z),(\omega,x_{2n})\big)(\omega,u), \\ F\big((\omega,x_{2n}),G(\omega,z)\big)(\omega,u)\end{matrix}\right\}$$

$$= g^{-1}\{g[F((\omega,z),(\omega,x_{2n}))(\omega,u)] + g[F((\omega,x_{2n}),G(\omega,z))(\omega,u)]\}$$

$$= g^{-1}\left\{\begin{matrix} g[F((\omega,z),(\omega,x_{2n}))(\omega,u)] \\ +g[F(T(\omega,x_{2n-1}),G(\omega,z))(\omega,u)]\end{matrix}\right\}$$

$$= g^{-1}\left\{\begin{matrix} g[F((\omega,z),(\omega,x_{2n}))(\omega,u)] \\ +\alpha g\left[F((\omega,x_{2n-2}),(\omega,z))\left(\frac{(\omega,u)}{\alpha}\right)\right]\end{matrix}\right\}$$

$$\geq \lim_{n\to\infty} g^{-1}\left\{\begin{matrix} g[F((\omega,z),(\omega,x_{2n}))(\omega,u)] \\ +\alpha g[F((\omega,z),(\omega,x_{2n-1}))((\omega,u))/\alpha)]\end{matrix}\right\} = \omega$$

Using (3.1(a)) and (3.1(b)) we get $G(\omega,z) = (\omega,z)$.
Again,

$$F((\omega,z),G(\omega,z))(\omega,u) \geq T\left\{\begin{matrix} F\big((\omega,z),(\omega,x_{2n+1})\big)(\omega,u), \\ F((\omega,x_{2n+1}),T(\omega,z))(\omega,u)\end{matrix}\right\}$$

$$= g^{-1}\left\{\begin{matrix} g[F((\omega,z),(\omega,x_{2n+1}))(\omega,u),] \\ +g[F((\omega,x_{2n+1}),T(\omega,z))(\omega,u)]\end{matrix}\right\}$$

$$= g^{-1}\{g[F((\omega,z),(\omega,x_{2n+1}))(\omega,u)] + g[F(G(\omega,x_{2n}),T(\omega,z))(\omega,u)]\}$$

$$= g^{-1}\left\{\begin{matrix} g[F((\omega,z),(\omega,x_{2n+1}))(\omega,u)] \\ +\alpha g[F((\omega,x_{2n}),T(\omega,z))((\omega,u)/\alpha)]\end{matrix}\right\}$$

$$\geq \lim_{n\to\infty} g^{-1}\left\{\begin{matrix} g[F((\omega,z),(\omega,x_{2n+1}))(\omega,u)] \\ +\alpha g[F((\omega,x_{2n}),T(\omega,z))((\omega,u)/\alpha)]\end{matrix}\right\} = \omega$$

Thus (ω,z) is common fixed point of G and T.
In order to show that (ω,z) is the only common fixed point of G and T, if possible let (ω,w) be any other common fixed point of G and T
We have from (3.1(a))

$$F((\omega,z),(\omega,w))(\omega,u) = F(G(\omega,z),T(\omega,w))(\omega,u)$$

$$g\{F((\omega,z),(\omega,w))(\omega,u)\} = g\{F(G(\omega,z),T(\omega,w))(\omega,u)\}$$

$$\leq \alpha g \max\left\{\begin{matrix} F((\omega,z),(\omega,w))\left(\frac{(\omega,u)}{\alpha}\right), F((\omega,z),G(\omega,z))\left(\frac{(\omega,u)}{\alpha}\right), \\ F((\omega,w),T(\omega,w))\left(\frac{(\omega,u)}{\alpha}\right), F((\omega,w),G(\omega,z))\left(\frac{(\omega,u)}{\alpha}\right)\end{matrix}\right\}$$

$$\leq \alpha g\left\{F((\omega,z),(\omega,w))\left(\frac{(\omega,u)}{\alpha}\right)\right\}$$

Therefore $g\{F((\omega,z),(\omega,w))(\omega,u)\} \leq \alpha g\left\{F((\omega,z),(\omega,w))\left(\frac{(\omega,u)}{\alpha}\right)\right\} < g\left\{F((\omega,z),(\omega,w))\left(\frac{(\omega,u)}{\alpha}\right)\right\}$ since $\alpha < 1$.

This implies $F((\omega,z),(\omega,w))(\omega,u) \geq F((\omega,z),(\omega,w))\left(\frac{(\omega,u)}{\alpha}\right)$ since g is decreasing function.

This gives a contradiction, as $\frac{(\omega,u)}{\alpha} > (\omega,u)$ as $\alpha < 1$ and $F_{\alpha_{x,y}}(u)$ is non decreasing function.

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This implies $(\omega, z) = (\omega, w)$
This completes the proof.

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