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AVAILABILITY IMPROVEMENT OF A SYSTEM USING SUBSTITUTE SYSTEM

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Abstract In present paper repair and replacement policy of one unit system is considered with substitute system.Assuming failure and repair times as exponentially distributed, expressions for the mean time to system failure (MTSF) and the steady state availability for system are derived using linear first order differential equations. A particular case for the proposed system is discussed in which substitute system is not considered. Graphical comparison is also performed to observe the effect of the proposed system on Availability.

Key Words: Availability, Linear first order differential equation, Mean Time to System Failure, Reliability, Steady State Availability.

Introduction-

Competition exists in every field, to keep ahead a major challenge is availability improvement of a system, as less availability has negative impact. People often use "availability" and "reliability" interchangeably. In fact, however, the two terms are related but have distinct meanings. Reliability (as measure of the mean time between system failures, or MTBF) is one of two key components of availability. The other is the mean time required to repair a given system when it fails, or MTTR. The formula for availability is as follows:

Availability = MTBF / (MTBF + MTTR)

We can have example of a power supply system which is highly reliable, because it rarely experiences downtime, but not highly available because it has a high mean time to repair.

Many authors studied single unit systems, cold standby systems, warm standby systems etc. to improve reliability. But very less attention was paid to improve availability. The present study is an effort to improve availability by introducing a substitute system. None of the researchers

considered the concept of substitute system facility which has been used in the present paper, here the system consists of a single unit. It has been observed that the unit in the system may fail due to repairable or some irreparable fault which has to be replaced.On the failure of the operative unit, it is repaired or its component is replaced witha new one according as it is repairable or irreparable. If repair or replacement of a unit can be completed in small time then repair or replacement will be continued and the system is brought back to the operative condition. But if repair or replacement is taking more time than, some other substitute system (may be cheaper or on rent) is called for continuation of operation with guarantee of failure free operation to resume the desired operation. There may be a short period of downtime but the impact is much less than it would be otherwise. The substitute system is returned back only when the original system starts working as good as new after repair or replacement.

Material and Methods

In this study system is analyzed by making use of Linear first order differential equationsand have obtained measures of system effectiveness such as Mean time to system failure and Availability.

Notations

O = Operative unit.

S = Substitute system.

F_{ur} = Failed unit is under repair.

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F_{urp} = Failed unit is under replacement.
a = Constant failure rate of unit.
b = Constant rate of failed unit isunder repair.
c = Constant rate of failed unit is under replacement.
d = Rate of restoring to working condition of system after repair or replacementwhile connected to substitute system.
f = Repair rate of a unit.
g = Replacement rate of a unit.
h = Rate of connecting substitute system.

States of the System

The system may be in one of the following states:

S_0 (O), S_1 (F)S_2 (F_{ur}), S_3 (F_{urp}), S_4 (S)

The transition diagram is shown in *Figure* 1

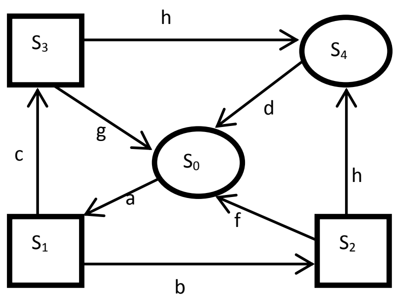

Figure 1

Mean Time To System Failure (Mtsf)

The mean time to system failure (MTSF) for the proposed system will be evaluated using the linear first order differential equations. Let P_i (t) is the probability that the system at time t, (t ≥ 0) is in state S_i. Let P (t) denote the probability row vector at time t, the initial conditions for this problem are:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0)]$$
$$= [1, 0, 0, 0, 0] \quad \dots (1)$$

By employing the method of linear first order differential equations, we obtain the following differential equations:

$$\frac{dP_0}{dt} = -aP_0 + fP_2 + gP_3 + d\,P_4$$

$$\frac{dP_1}{dt} = aP_0 - (b+c)P_1$$

$$\frac{dP_2}{dt} = bP_1 - (f+h)P_2$$

$$\frac{dP_3}{dt} = cP_1 - (g+h)P_3$$

$$\frac{dP_4}{dt} = hP_2 - dP_4 + hP_3$$

This can be written in the matrix form as:

P *= Q P,

Where

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$$Q = \begin{bmatrix} -a & 0 & f & g & d \\ a & -(b+c) & 0 & 0 & 0 \\ 0 & b & -(f+h) & 0 & 0 \\ 0 & c & 0 & -(g+h) & 0 \\ 0 & 0 & h & h & -d \end{bmatrix}$$

To calculate the MTSF, we take the transpose of the matrix Q and delete the rows and columns for the absorbing state. The new matrix is called (A). The expected time to reach an absorbing state is calculated from

$$E[T_{P(0) \rightarrow P(absorbing)}] = P(0)(-A^{-1})\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad ...(2)$$

where

$$A = \begin{bmatrix} -a & 0 \\ d & -d \end{bmatrix}$$

We obtain the following expression for MTSF on solving equation (2).

$$MTSF = \frac{1}{a},$$

Availability Analysis Of The System

The initial condition for this problem is same as for the reliability case i.e.

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0)] = [1, 0, 0, 0, 0]$

This can be written in the matrix form as:

$P^* = Q P$,

Where

$$\begin{bmatrix} P_0^* \\ P_1^* \\ P_2^* \\ P_3^* \\ P_4^* \end{bmatrix} = \begin{bmatrix} -a & 0 & f & g & d \\ a & -(b+c) & 0 & 0 & 0 \\ 0 & b & -(f+h) & 0 & 0 \\ 0 & c & 0 & -(g+h) & 0 \\ 0 & 0 & h & h & -d \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

Let t be the time to failure of the system. Then steady-state availability is given by

$A_t(\infty) = P_0(\infty) + P_4(\infty)$

In the steady-state situation, the derivatives of the state probabilities become zero. That is

$QP(\infty) = 0$

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Then above matrix becomes

$$\begin{bmatrix} -a & 0 & f & g & d \\ a & -(b+c) & 0 & 0 & 0 \\ 0 & b & -(f+h) & 0 & 0 \\ 0 & c & 0 & -(g+h) & 0 \\ 0 & 0 & h & h & -d \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

On substituting the normalizing condition $\sum_{i=0}^{4} P_i(\infty) = 1$, in any one of the redundant rows of above matrix and on solving, the solution provides the steady-state probabilities $P_0(\infty)$, $P_1(\infty)$,..., $P_4(\infty)$. Expression for steady-state availability is thus

$$A(\infty) = \frac{N_1}{D_1} \text{ where}$$

$$N_1 = d(ch^2 + bh^2) + g(dch + bdh + df(c+b)) + df(ch + bh) + a(ch^2 + bh^2) + abgh + acfh$$

$$D_1 = a(dh^2 + cdh + bdh + ch^2 + bh^2) + d(ch^2 + bh^2) + f(adh + adc + ach + dch + dbh) + g[adh + adb + abh + d(ch + bh) + (dc + bd + da)f].$$

Comparison With Particular Case

When substitute system is not used then by availability expression can be obtained as

$$A(\infty) = P_0(\infty)$$

$$A(\infty) = \frac{(c+b)fg}{((c+b+a)f + ab)g + acf}$$

For the model comparison, the following set of parameters values are fixed for consistency

$0.01 \le a \le 0.06$, b= 0.2, c= 0.2, d= 0.5, f= 0.4, g= 0.4, h=0.1

a	Availability with substitute system	Availability without substitute system
0.1	0.791476407914764	0.666666666666667
0.2	0.638297872340425	0.5
0.3	0.541062801932367	0.4
0.4	0.473856209150327	0.333333333333333
0.5	0.424628450106157	0.285714285714286
0.6	0.387016229712859	0.25

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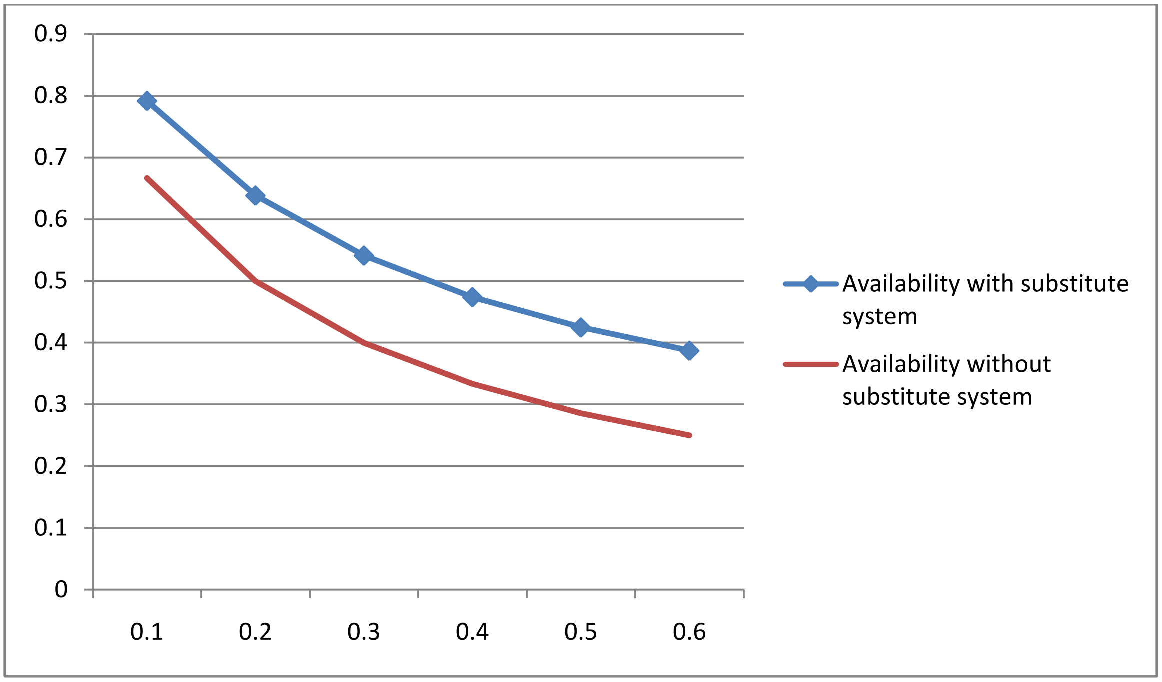

Figure 2

It is apparent from the above table and *Figure 2* that the availability is improved by incorporating thefacility of substitute system. These tend to suggest that system with substitute system is better than the other systems.

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