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AN INVENTORY MODEL WITH PRICE BREAK

Vandana Malviya[1]
Research Scholar, Mewar University, Chittorgarh, Rajasthan, India

Dr. Vijay Kr. Agarwal[2]
Department of Mathematics, Modi University of Science and Technology, Faculty of Engineering and Technology, Laxmangarh, Sikar, Rajasthan, India

Abstract : In this paper, we discuss a situation in which cost of the item charged is less if the quantity of items purchased is more than a particular quantity. This is generally referred to as quantity discount. Alongwith quantity discount transportation cost has also been taken into consideration explicitly. We assume that the cost of transportation is fixed for a transport mode, say truck, whether it is fully loaded or partially loaded. Transportation cost happens to be a discrete function of the lot-size. For the transportation, cost as given in this paper, which also exists in real world, should be considered along with the inventory cost so as to obtain total minimum cost of transportation and inventory.

Key words : Transportation, inventory, quantity discount

Introduction

The control and maintenance of inventories of physical goods is a problem common to all enterprises in any sector of a given economy. For example, inventories must be maintained in agriculture, industry, retail establishment, military etc. The fundamental reason for maintaining inventory is that it is either physically impossible or economically unsound to have goods arrive in a given system precisely as and when demand for them occurs.

Although inventory problem are as old as history itself, it has only been since the turn of the century that any attempt has been made to employ analytical technique in studying these problems. The earliest derivation of what is often called the simple lot-size formula was obtained by Ford Harris and R. H. Wilson in 1915. This formula was applicable to very few inventory systems and modifications according to the practical situation were required. Researchers came forward and various extensions to the simple lot-size model used in practice were given.

In this chapter we discuss a situation in which cost of the item charged is less if the quantity of items purchased is more than a particular quantity. This is generally referred to as quantity discount. Alongwith quantity discount transportation cost has also been taken into consideration explicitly. We assume that the cost of transportation is fixed for a transport mode, say truck, whether it is fully loaded or partially loaded. Transportation cost happens to be a discrete function of the lot-size. Hadley and Whitin (1963) have given the general formula for 'n' numbers of quantity discounts. Gupta (1992) has discussed a joint transport-inventory model in which transportation cost with simple inventory model has been considered. Gupta and Agarwal (1996) have extended Gupta's model by relaxing restrictive assumptions and the algorithm has been simplified for solving numerical problems. Others to study about joint

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inventory-transport model were Anily and Federgruen (1990), Baumol and Vinod (1970), Buffa and Reynolds (1977), Constable and Whybark (1978), Das (1974), Langley (1981) and Larson (1988), O.K.Gupta (1992), P.N.Gupta and V.K.Agarwal (1996), Nozick, L.K. (2000), Q.H.Zhao, S.Y.Wang, K.K.Lai & G.P.Xai (2004), Cecere, L., Hofman, D., 2007, Rieksts, B.Q., Ventura, J.A. (2010).

Model Development

Assumptions and Notations used in this model are as follows:

Assumptions:

1. Demand rate is uniform and constant.
2. Rate of replenishment is infinite.
3. Shortages are not allowed.
4. Lead time is constant.
5. Cost of an item purchased is fixed if the quantity purchased is less than or equal to a particular number of items. Cost is reduced if the quantity purchased is more than that particular number of items.
6. The transportation cost is constant for a truck load (of a given capacity), even though the quantity transported is less than a truck load.
7. It is the liability of the buyer to pay for the transportation.

Notations used are as follows:

A= Set-up cost per set-up.

D= Demand rate.

i= Cost of holding one unit of currency for a unit time.

C_a=Cost of item before quantity discount.

C_b=Cost of item after quantity discount.

C_2= Transportation cost upto one truck load.

K= Capacity of the truck (in units).

P= Number of units from which quantity discount becomes applicable.

q = Lot-size.

q_1^* = Lot-size for which total average cost of inventory is minimum when cost per item is C_a.

q_2^* = Lot-size for which total average cost of inventory is minimum when cost per item is C_b.

Q_1^* = Lot-size for which total average cost of transportation and inventory is minimum when cost per item is C_a.

Q_2^* = Lot-size for which total average cost of transportation and inventory is minimum when cost per item is C_b.

M = Number of trucks used to carry the load of required lot-size.

M_1= Number of trucks used to carry the load of lot-size q_1^*.

M_2= Number of trucks used to carry the load of lot-size q_2^*.

M_p= Number of trucks used to carry the load of lot-size p.

$\lceil x \rceil$ = Lowest integer greater than or equal to x.

$f(q)$ = Total average cost of transportation and inventory per unit time for lot-size q.

Mathematical Analysis

For the moment we do not take transportation cost into an account and let the general problem be of the type given as under.

If $0 < q < P$ then cost of unit item is C_a and if $p \le q$ then the cost of unit item is C_b. From Harris-Wilson formula we have $q_1^* = \left\{\frac{2AD}{iC_a}\right\}^{1/2}$ as the lot-size for which total average cost of inventory per unit time is minimum, when cost of unit item is C_a. Similarly we have $q_2^* = \left\{\frac{2AD}{iC_b}\right\}^{1/2}$ as the lot-size for which total average cost of inventory per unit time is minimum, when cost of unit item is C_b.

Under different situations we may have different cases as given below.

(1) $q_1^* < P$ and $q_2^* < P$.

(2) $q_1^* < P$ and $q_2^* \ge P$.

(3) $q_1^* \ge P$ and $q_1^* \ge P$.

We can calculate total minimum average cost of transportation and inventory per unit time from the formula $Q_1^* = \left\{\frac{2(A+C_2M_1)D}{iC_a}\right\}^{1/2}$ (or $Q_2^* = \left\{\frac{2(A+C_2M_2)D}{iC_a}\right\}^{1/2}$) taking $M_1 = {q_1^*}/{K}$ (or $M_2 = {q_2^*}/{K}$), but if $Q_1^* > M_1K$ (or $Q_2^* > M_2K$) then Q_1^* (or Q_2^*) is

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not feasible and we analysis the problem as follows.

(1) For transportation cost $\frac{C_2MD}{q}$, we observe that it suddenly jumps up as q becomes slightly greater than MK, then it decreases continuously till q becomes $(M+1)K$, and so on. Thus C_2MD/q is minimum at $q = MK$ and this minimum remains same for all M. In other words C_2MD/q increases as we deviate from the minimum occurs at $q = MK, M = 1, 2, \dots \dots$

(2) We know that the cost $\frac{AD}{q} + \frac{iC_aq}{2}$ (or $\frac{AD}{q} + \frac{iC_bq}{2}$) continuously decreases upto q_1^* (or q_2^*) and then increases continuously, therefore total minimum average cost per unit time for different cases can be obtained as follows.

Case 1: When $q_1^* < P$ and $q_2^* < P$.

In this case q_1^* is feasible and q_2^* is not feasible, therefore sum of the average set up cost and average holding cost per unit time is minimum at lot size q_1^*, when cost of unit item is C_a, and when the cost of unit item is C_b lot size which refers to minimum sum of average set up cost and average holding cost per unit time is "P".

From the points noted above we can easily conclude that total minimum average cost of transportation and inventory per unit time corresponds to the lot size q_1^*, if it is between points $q_1^*M_1 (= M_1K)$ and $q_1^*M_1 - 1 (= (M_1 - 1)K)$ otherwise it corresponds to either of $q_1^*M_1$ and $q_1^*M_1 - 1$, when the cost of unit item is C_a. Total minimum average cost of transportation and inventory per unit time lies at Q_2^* if it is feasible, otherwise at $q_2^*M_P (= M_PK)$ when the cost of unit item is C_b.

Finally total minimum average cost per unit time obtained for the case when cost of unit item is C_a, and for the case when cost of unit item is C_b are compared and lesses of the two gives the optimum lot-size.

Case 2: When $q_1^* < P$ and $q_2^* \geq P$.

In this case both q_1^* as well as q_2^* are feasible and so, as discussed in case 1, optimum lot-size, when the cost of unit item is C_a, optimum lot-size, when the cost of unit item is C_a, is Q_1^* if it lies between the points $q_1^*M_1 (= M_1K)$ and $q_1^*M_1 - 1 (= (M_1 - 1)K)$, otherwise at $q_1^*M_1$ or $q_1^*M_1 - 1$. When the cost of unit item is C_b, optimum lot-size is Q_2^* if it lies between $q_2^*M_2 (= M_2K)$ and $q_2^*M_2 - 1 (= (M_2 - 1)K)$, otherwise at $q_2^*M_2$ or $q_2^*M_2 - 1$. Again, lesser of the minimum costs obtained from the lot-sizes, are the optimal cost.

Case 3: When $q_1^* \geq P$ and $q_2^* \geq P$.

In this case q_1^* is not feasible and q_2^* is feasible, therefore $q_1^*M_P (= M_PK)$ is calculated when the cost of unit item is 'C_a'. If 'C_b' is the cost of unit item, we first calculate Q_2^*. If it is not feasible we calculate the cost corresponding to $q_2^*M_2 (= M_2K)$ and $q_2^*M_2 - 1 (= (M_2 - 1)K)$ and choose the smaller one. Final optimum cost is obtained by comparing the two minimum average costs due to lot-sizes for the unit costs 'C_a' and 'C_b' respectively.

Conclusion

From the given examples, we can observe that optimum lot-size, when transportation cost of the type given in this paper is included, comes out to be different than the lot-size when transportation cost is not included. Thus for the transportation, cost as given in this chapter, which also exists in real world, should be considered along with the inventory cost so as to obtain total minimum cost of transportation and inventory.

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