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A HEURISTIC METHOD TO MINIMIZE THE RANGE OF LATENESS IN A SPECIAL CLASS OF n-JOBS, 2-MACHINE SEQUENCING PROBLEM WITH DUE-DATES, TRANSPORTATION TIMES AND EQUIVQLENT JOB FOR BLOCK OF JOBS

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Abstract : In the present paper a heuristic method has been developed to minimize the range of lateness in a sequencing/scheduling problem of n-job, 2-machine with due-date and equivalent job for block of jobs. Transportation times of jobs from first machine to second machine have also been included in the problem. The heuristic method is based on three theorems. The paper investigates some conditions to be imposed on processing times and transportation times. The criterion to find optimal sequence is the minimization of range of lateness. In the paper a rule to find equivalent job and its processing times for jobs in block has been determined. The work has been supported by a numerical example.

Key words : Range of Lateness, Due-dates, Sequencing/Scheduling, Operation Research.

Introduction

The problem of n-jobs, on a single machine with due-date was studied by many researchers with different objectives of finding optimal sequence including minimization of flow time, total lateness, waiting time variance, variance of flow time etc. Sushil Gupta and Tapan Sen [4] studied the problem of n-jobs a single machine with due-date with the objective of minimizing the difference between maximum and minimum job lateness. Sushil and Tapan Sen [4] justified that such problems are important in real life whenever it is desirable to give equal treatment to all customers (jobs).

They denoted t_i , c_i and d_i as processing time, completion time due –date of jobs $i, i = 1,2,3,\dots,n$

Lateness of job $i, L_i = c_i - d_i \;\; i = 1,2,\dots,n.$

Slack time of job $i = d_i - t_i$.

For a given sequence s, the value of objective function

$$z(s) = max\{L_i\} - min\{L_i\}, i = 1,2,\dots,n$$

The optimal sequence defined by them is which minimizes L(s) or the sequence which minimizes range of lateness is optimal sequence. Sushil and Tapin sen [4] proved in their paper that a sequence in which jobs are arranged as per MST rule, is optimal sequence, if jobs are also in EDD order. That is, the sequence in which jobs are in both MST and EDD minimizes the range of lateness.

The concept of minimization of range of lateness in n-jobs, one machine with due-date due to Sushil and Tapin [4], was extended to n-jobs, 2-machine sequencing problem with due-date by Ikram and Tahir[5]. They studied n-jobs, 2-machine problem with due-date satisfying some condition imposed on processing time of jobs of two machines A and B in the order AB with the objective to minimize the range of lateness. They denoted A_i, B_i as processing time of jobs $i, i = 1,2,3,\dots,n$ on machine A and B respectively.

d_i = due-date of jobs $i. i = 1,2,3,\dots,n$

c_i = completion time of jobs i on machine B

Lateness of job$i, L_i = c_i - d_i \;\; i = 1,2,\dots,n.$

Slack time of job $i = d_i - B_i$

Ikram and Tahir [5] proved that if$Max\{A_i\} - Min\{B_i\}$

Then a sequence in which jobs are arranged according to MST rule, minimizes range of lateness if jobs are also in EDD order in that sequence.

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Ikram and Tahir [6] introduction the concept of transportation time due to Maggu and Das [2] in the problem studied by Ikram and Tahir[5].
Further, G.S.Sodi and Kanwarjit Singh [7] introducted the concept of equivalent job for jobs in blocks due to Maggu and Das [3] , in the problem of Ikramine and Tahir[5].

In the present paper both the concept of transportation time and equivalent job for block of jobs have been introduced simultaneously with the objective to minimize range of lateness in n-job, 2-machine sequencing problem, satisfying some conditions imposed on transportation times and processing times.

Formulation Of Problem

Arearras	Machine A(A_1)	Transportation time (t_i)	Machine B (B_i)	Due-date (d_i)
1	A_1	t_1	B_1	d_1
2	A_2	t_2	B_2	d_2
3	A_3	t_3	B_3	d_3
4	N			
5	N			
N	N			
n	A_n	t_n	B_n	d_n

Where A_i, B_i are processing time of jobs $i. i = 1,2,3, \dots, n$ on the machine A and B respectively; of jobs t_i and d_i are transportation time and due-date of jobs i,i $= 1,2,3, \dots, n$.
Two jobs are forming a block.
The problem is to determine a sequence (optimal sequence) which minimizes the range of lateness.

Let
c_i = completion time of jobs i on machine B
Lateness of job$i, L_i = c_i - d_i \;\; i = 1,2, \dots, n$.
Slack time of job $i = d_i - B_i$
The value of objective function for a given sequence s is

$$z(s) = {}^{max}{}_i\{L_i\} - {}^{min}{}_i\{L_i\}$$

The optimal sequence is the sequence for which z(s) is minimum or optimal sequence is that which minimize the range of lateness.

Sugessted Conditions

Max$\{A_i + t_i\} \le Min\{B_i\}$
..........(a)

These conditions are different from the conditions used in the paper [5].

Theorem I. Let[1],[2],[3],....,[i-1],[i],....,[n] be a sequence of jobs where[i] denotes the job which is sequenced at place. Then the completion time of job[i] on machine B in the above stated problem satisfying condition given in (a), is given by

$$A_{[1]} + t_{[1]} + \sum_{j=1}^{[i]} B_j$$

Proof. In the sequence [1][2][3],...[n], the completion time of job[1] on machine $B = A_{[1]} + t_{[1]} + B_{[1]}$.
Completion time of job [2] on machine $A = A_{[1]} + A_{[2]}$.

Here it is to be noted that process of transportation of job[1] after its completion on machine A and processing of job[2] on machine A can go simultaneously. Therefore, job[2] will be ready to be processed on machine B at $A_{[1]} + A_{[2]} + t_2$ whereas machine B is free at the time $A_{[1]} + t_{[1]} + B_{[1]}$

Now $A_{[2]} + t_{[2]} \le B_{[1]}$, using (a)
or $A_{[2]} + t_{[2]} \le t_{[1]} + B_{[1]}$
or $A_{[1]} + A_{[2]} + t_{[2]} \le A_{[1]} + t_{[1]} + B_{[1]}$
So, $Max\{A_{[1]} + A_{[2]} + t_{[2]}, A_{[1]} + t_{[1]} + B_{[1]}\} = A_{[1]} + t_{[1]} +$
$B_{[1]}$ Job [2] will go on machine B at the time= $A_{[1]} + t_{[1]} + B_{[1]}$
and will complete on B at the time $= A_{[1]} + t_{[1]} + B_{[1]} + B_{[2]}$

Job[3] is completed on machine A at the time

$$= A_{[1]} + A_{[2]} + A_{[3]}$$

and will be ready for its process on machine B at $= A_{[1]} + A_{[2]} + A_{[3]} + t_{[3]}$

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$Max\{A_{[1]} + A_{[2]} + A_{[3]} + t_{[3]}, A_{[1]} + t_{[1]} + B_{[1]} + B_{[2]}$
$= A_{[1]} + t_{[1]} + B_{[1]} + B_{[2]}$

$\therefore$ Machine B will take job [3] for processing at

$+t_{[1]} + B_{[1]} + B_{[2]}$

Hence completion time of job [3] on machine B
$= A_{[1]} + t_{[1]} + B_{[1]} + B_{[2]} + B_{[3]}$

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$Max\{A_{[1]} + A_{[2]} + \dots + A_{[i]} + t_{[i]}, A_{[1]} + t_{[1]} + B_{[1]} + B_{[2]} + \cdots + B_{[i-1]}\} = A_{[1]} + t_{[1]} + B_{[1]} + B_{[2]} + \cdots + B_{[i-1]}$

$\therefore$ job [i] will complete at machine B at

$= A_{[1]} + t_{[1]} + B_{[1]} + B_{[2]} + \cdots + B_{[i-1]} + B_{[i]}$

or $= A_{[1]} + t_{[1]} + \sum_{j=1}^{[i]} B_j$.

Proved

Theorem 2. For the above stated problem satisfying conditions give in (a) without job block, the optimal sequence (minimizing range of lateness) is that in which jobs are both in MST and EDD.

Proof . From theorem (I),the completion time of job [i] on machine B is

$= A_{[1]} + t_{[1]} + \sum_{j=1}^{[i]} B_j$.

Let $A_{[1]} + t_{[1]} = A'_{[1]}$, then

The completion time of job[i] on machine B= $A'_{[1]} + \sum_{j=1}^{[i]} B_j$. ………..**(b)**

Ikram and Tahir [5] proved in their study that if completion time of job[i] on machine B is

$= A_{[1]} + \sum_{j=1}^{[i]} B_j$ ………..**(c)**

Then the sequence minimizes range of lateness in which jobs are in MST and EDD both terms given in *b* and *c* are same. Therefore, for the above stated problem also, that sequence minimizes the range of lateness in which jobs are both in MST and EDD i.e. the optimal sequence is that in which jobs are in MST and EDD.

Determination Of Equivalent Job For Block Of Jobs

Theorem 3. If jobs forming the block have processing times $A_{\alpha_k}, A_{\alpha_{k+1}}$ on machine A and $B_{\alpha_k}, B_{\alpha_{k+1}}$ on machine B. Let β be equivalent job for jobs in block, then processing times of job β on machine A and B are given by $A_\beta = A_{\alpha_k}$

$B_\beta = B_{\alpha_k} + B_{\alpha_{k+1}} - A_{\alpha_{k+1}}$

Proof. Maggu and Das [3] proved in their paper that the processing of equivalent job β on machine A and B can be calculated by using following formulae. The replacement of jobs in block by β does not increase the completion time $= A_{\alpha_k} + A_{\alpha_{k+1}} - \min\{B_{\alpha_k}, A_{\alpha_{k+1}}\}$

and $B_\beta = B_{\alpha_k} + B_{\alpha_{k+1}} - \min\{B_{\alpha_k}, A_{\alpha_{k+1}}\}$

But $Max\{A_i + t_i\} \le min\{B_i\}$ $i = 1,2,\dots,n$

$\therefore \min\{B_{\alpha_k}, A_{\alpha_{k+1}}\} = A_{\alpha_{k+1}}$

$\Rightarrow A_\beta = A_{\alpha_k} + A_{\alpha_{k+1}} - A_{\alpha_{k+1}}$

$\Rightarrow A_\beta = A_{\alpha_k}$ ……….**(d)**

now $B_\beta = B_{\alpha_k} + B_{\alpha_{k+1}} - \min\{B_{\alpha_k}, A_{\alpha_{k+1}}\}$

but $max\{A_i + t_i\} \le min\{B_i\}$

$\Rightarrow max\{A_i\} \le min\{B_i\}$

$\Rightarrow \min\{B_{\alpha_k}, A_{\alpha_{k+1}}\} = A_{\alpha_{k+1}}$

$\therefore B_\beta = B_{\alpha_k} + B_{\alpha_{k+1}} - A_{\alpha_{k+1}}$ …………**(e)**

From (d) and (e), the theorem (3) is proved.

On the basis of above 3-theorem, the following Heuristic Method is developed for the problem defined in this paper with the objective to minimize the range of lateness.

Heuristic Method

Step I : Verify the condition $Max\{A_i + t_i\} =\le Min\{B_i\}$ i =1,2,3,…….n.

Step II : Determine the equivalent job β for block of jobs and its processing times on machine A and B using(d) and (e).

Step III : Obtain modified problem replacing block of jobs by equivalent job β and its processing times as calculate in step II. Let t_{α_k} and $t_{\alpha_{k+1}}$ are transportation time and $d_{\alpha_k} + d_{\alpha_{k+1}}$are due-date for jobs in block, then use

$t_\beta = t_{\alpha_k} + t_{\alpha_{k+1}} d_\beta = \min\{d_{\alpha_k}, d_{\alpha_{k+1}}\}$,

Step IV : Arrange the jobs according to MST rule and obtain sequence s.

Step V : If jobs in sequence obtained in Step IV are also in EDD order, then s is optimal or next to optimal sequence minimizing the range of lateness.

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Numerical Example

	Machine A(A_i)	Transportation time (t_i)	Machine B (B_i)	Due-date (d_i)
1	2	1	20	30
2	4	3	**16**	32
3	10	5	22	25
4	8	4	30	43
5	7	2	25	29
6	13	3	17	34
7	12	4	19	44

Jobs 2 and 6 are in block. The problem is to determine/next to optimal sequence minimizing range of late less.

Solution

Step I: $Max\{A_i + t_i\} = 16$

$Min\{B_i\} = 16$

$\therefore \quad Max\{A_i + t_i\} \le min\{B_i\}$.

Step II: Equivalent job β for jobs 2 and 6. The processing times for β $\Rightarrow A_\beta = A_{\alpha_k} = 4$

$B_\beta = B_{\alpha_k} + B_{\alpha_{k+1}} - A_{\alpha_{k+1}} = 16 + 17 - 13 = 20$

Step III: The modified problem replacing job 2 and 6 by equivalent job β and $d_\beta = Min\{d_2, d_6\} = Min\{32{,}34\} = 32$

$t_\beta = t_{\alpha_k} + t_{\alpha_{k+1}} = 3 + 3 = 6$

	Machine A(A_i)	Transportation time (t_i)	Machine B (B_i)	Due-date (d_i)
1	2	1	20	30
β	4	6	**20**	32
3	10	5	22	25
4	8	4	30	43
5	7	2	25	29
7	12	4	19	44

Step IV : slack time of jobs

$d_1 - B_1 = 30 - 20 = 10$

$d_\beta - B_\beta = 32 - 20 = 12$

$d_3 - B_3 = 25 - 22 = 3$

$d_4 - B_4 = 43 - 30 = 13$

$d_5 - B_5 = 29 - 25 = 4$

$d_7 - B_7 = 44 - 19 = 25$

Arrange jobs as per MST rule, we obtain the sequence 3,5,1,β,4,7.

Step V: In the sequence 3,5,1,β,4,7 we note that jobs also in EDD 3,5,1,β,4,7 Or 3,5,1,2,6,4,7 is optimal/next to optimal sequence for the given problem in which objective is to minimize range of lateness.

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